Realization of the unification height system between the Arab regions

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1. Introduction

- **UN-GGIM Arab States**

- **Working Group No. 3 (WG3) : Geodetic Reference Frame**

- Unified **geodetic** reference frames for Arab World with the same accuracy everywhere and fully consistent with the International Terrestrial Reference System and Frame (ITRS/ITRF).

- Unified **vertical** datum for Arab World with cm-level and fully consistent with the International Height Reference System (IHRS).

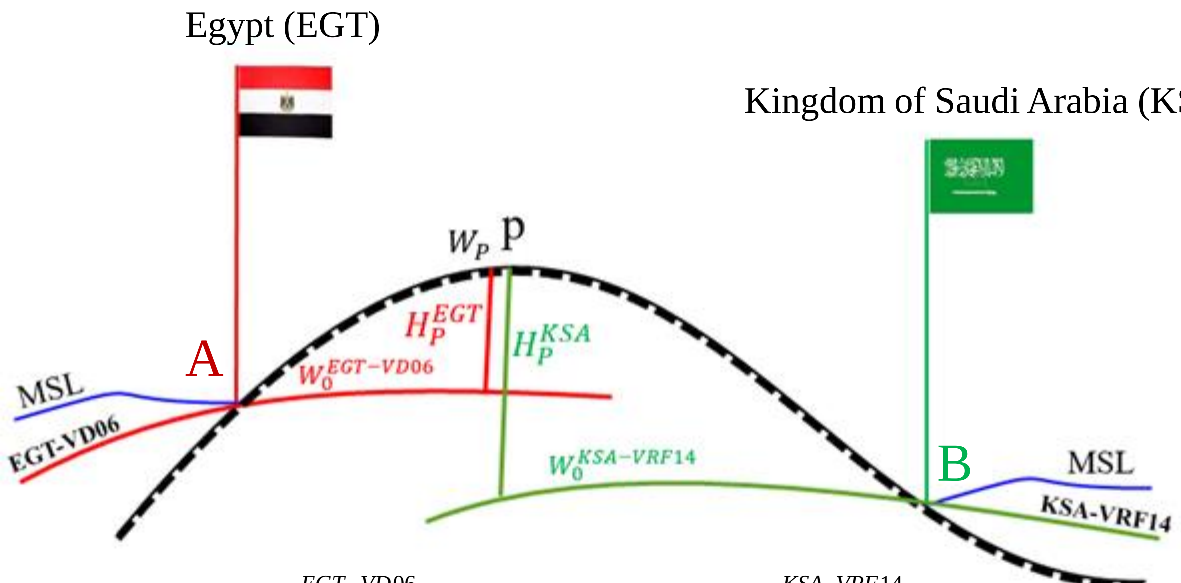
United Nations Global Geospatial Information Management (UN-GGIM) Arab States (22 Country) (Al-Kherayef, 2019)

- Work is underway to achieve the Arab Geodetic Reference Frame (ARABGRF), but the Arab Height Reference System (AHRS) has not yet been started (**Our Goal**).

1. Introduction

- **Existing height systems in Arab world (Today)**

- Refer to different levels (as many reference levels as reference tide gauges)



$$H_P^{EGT} = \frac{W_0^{EGT-VD06} - W_P}{\bar{g}_P} \neq H_P^{KSA} = \frac{W_0^{KSA-VRF14} - W_P}{\bar{g}_P} \quad (1)$$

Offsets $= \delta H_P^{EGT-KSA} \approx 6.0\ m$

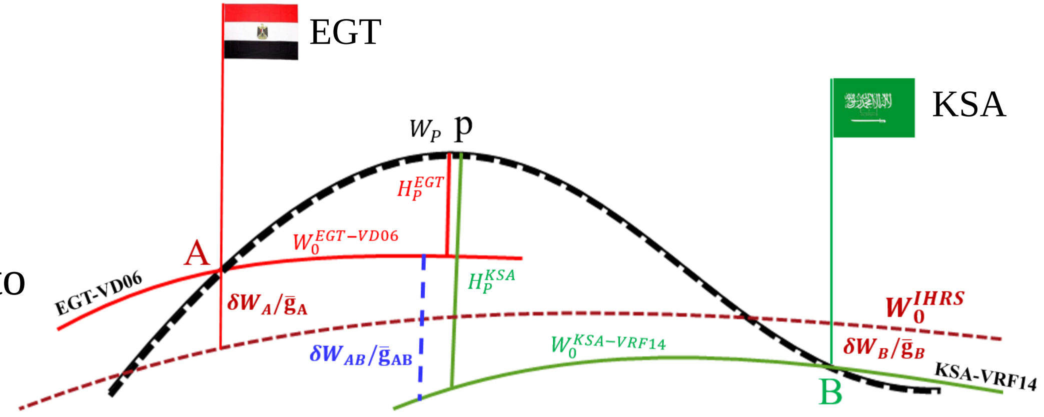
- Different types of **heights** (normal, orthometric …etc.)
- Omission of sea and land **vertical variations** with time;
- Variations at **borders** between datum zones

1. Introduction

▪ Unified ARAB Reference Level

- AHRS should be consistent with International Height Reference System (IHRS).

$$H_p = \frac{W_0^{IHRS} - W_P}{\bar{g}_P} = \frac{C_P}{\bar{g}_P} \quad (2)$$



- Now, **all** physical heights refer to one and the **same global level**.

$$H_P^{EGT} + (\delta W_A / \bar{g}_A) = H_P^{KSA} + (\delta W_B / \bar{g}_B) \quad (3)$$

- One of the **main components** for the definition of IHRS/AHRS is the **potential difference = geopotential numbers** for the reference stations:

$$C_P = W_o - W_p = -\delta W_P \quad (4)$$

$$W_o = const. = 62636853.353\ m^2 s^{-2}$$

Challenge: high-precision geopotential numbers or suitable computation methods for precise potential values.

1. Introduction

▪ Ways For Height System Unification

A. Conventional approach (Levelling + Gravimetry)

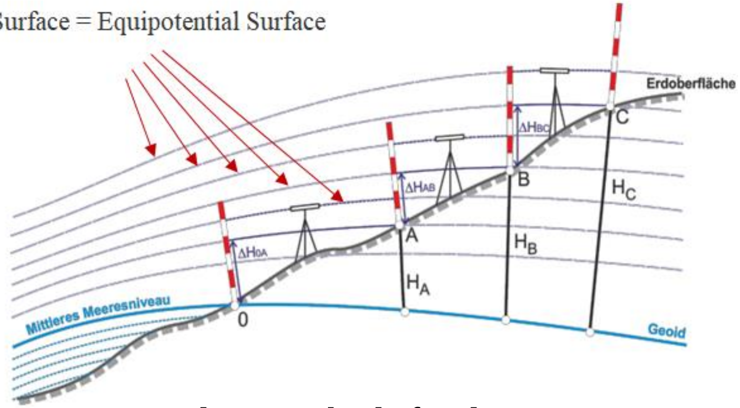
- It is mainly used for the realization of **local vertical datum**
- Suitable for the **transformation** of existing AHRS to IHRS

$$W_P = (W_0^{local} + \delta W) - C_P \quad ; \quad \delta W = W_0^{IHRS} - W_0^{local} \quad ; \quad C_P = \int_O^P g\, dH \qquad (5)$$

Defects

- Levelling errors (≈ cm) + **Integrations**.
- **Time** consuming and **repeated** measurements.
- **Hard** to realize perfectly in **mountains** area.
- Two points separated by sea **cannot be linked**.
- **Offsets** between different local height systems (≈ dm).

Level Surface = Equipotential Surface



Geodetic methods for determining geopotential and physical heights

(Sánchez et al., 2019)

1. Introduction

- **Ways For Height System Unification**

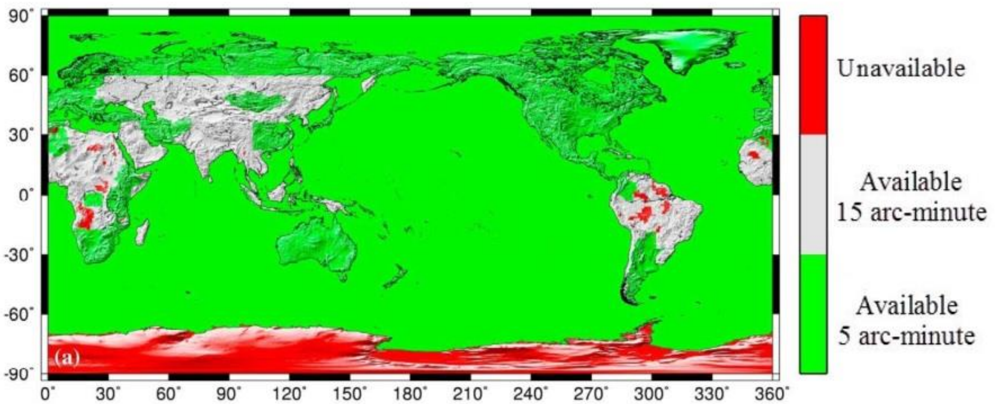
B. High Degree Global Gravity Models (GGMs)

- Represents the ideal way on **global scale**

$$C_P = W_0 - \left(\frac{GM}{r} \sum_{l=0}^{l_{max}} \sum_{m=0}^{l} \left[\frac{R}{r} \right]^l P_{lm}(\sin\varphi)(C_{lm}^W \cos m\lambda + S_{lm}^W \sin m\lambda) + \frac{1}{2}\omega^2 r^2 (\cos\varphi)^2 \right) \quad (6)$$

Defects

- Long **wavelength errors**
- **Different** Techniques (**Indirect**)
- Data (**heterogeneous + Gaps**)
- Satellite Missions (calibration+Integration)



Data availability used to develop EGM2008 model (Pavlis et al. 2012)

1. Introduction

- **Ways For Height System Unification**

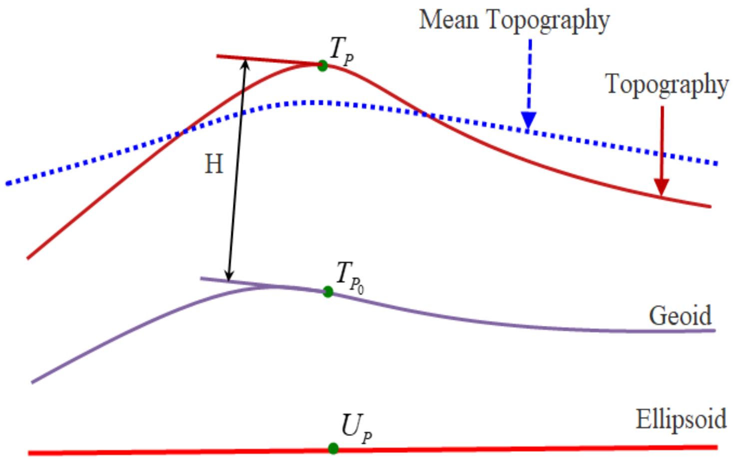
C. Disturbing Potential in combination with reference ellipsoid

It can provide a **global** solution for height unification

$$C_P = W_0 - W_P \ ; \ \ W_P = U_P + T_P \ \ ; \ T_P = T_{P(satellite-only)} + T_{P(residual)} + T_{P(terrain)} \quad (7)$$

Defects

- Determination of C_P depends on a series of **approximations** (Molodensky & Stokes.
- Use remove-restore technique, i.e. requires **previous information** of potential from various sources (global geopotential model, tide gauge data, gravity observation data, et al.)



Geometry of Earth's surface, geoid, and ellipsoid

i.e., different methodologies produce different potential values.

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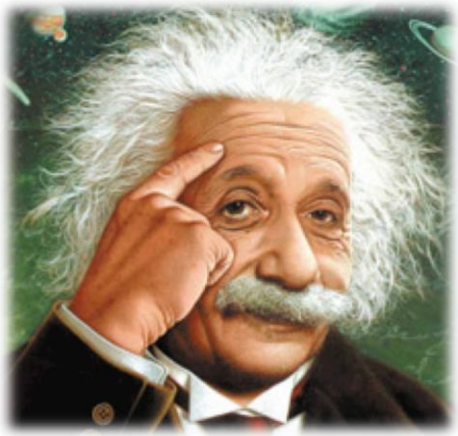
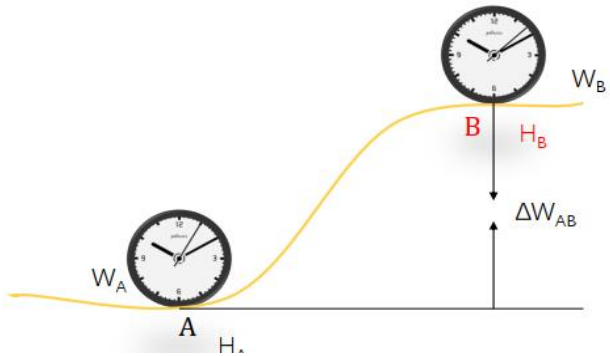
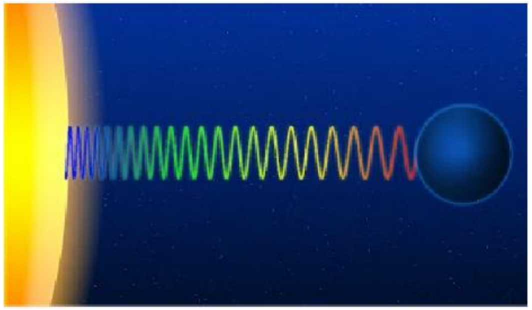
Relativity (Einstein 1915) : Predictions

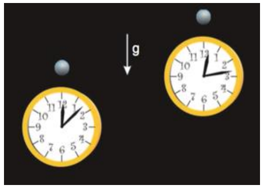






Gravitational redshift

$$\frac{d\tau_A}{dt} \approx 1 - \frac{W(t,\vec{x}_A)}{c^2} + O(c^{-4}) \quad (8)$$



(Bjerhammar 1985; Shen et. al, 1993; Shen, 1998)

- **Time comparison**: A clock runs **faster** at **higher** geopotential position than at **lower** position.
- **Frequency comparison**: The vibration frequency of the clock (oscillator) at **higher** geopotential position is **larger** than that at **lower** position.

2. Realization of Arab Height Reference System

Relativistic Geodesy

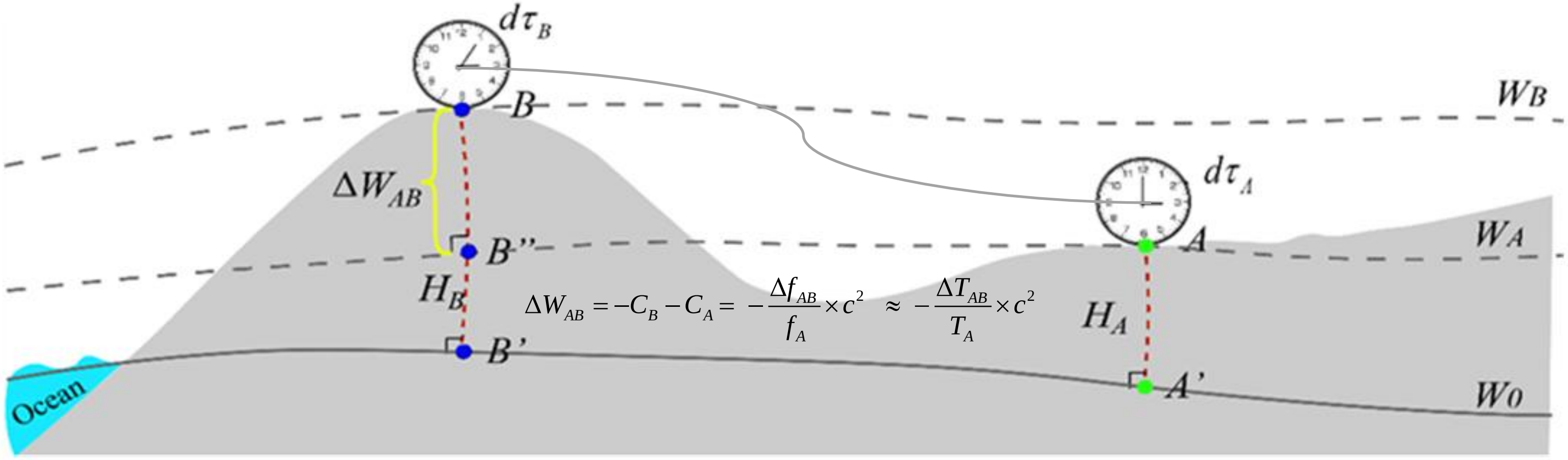
$$\frac{d\tau_A / dt}{d\tau_B / dt} = \frac{1 - c^{-2} W_A}{1 - c^{-2} W_B} \quad (9)$$

$$\Delta W_{AB}^{(T)} = W_B - W_A \approx c^2 \left(\frac{d\tau_B}{d\tau_A} - 1\right) \quad (10)$$

(Bjerhammar 1985; Shen et al. 2017 ;Wu & Müller 2020; Wu et al. 2020)

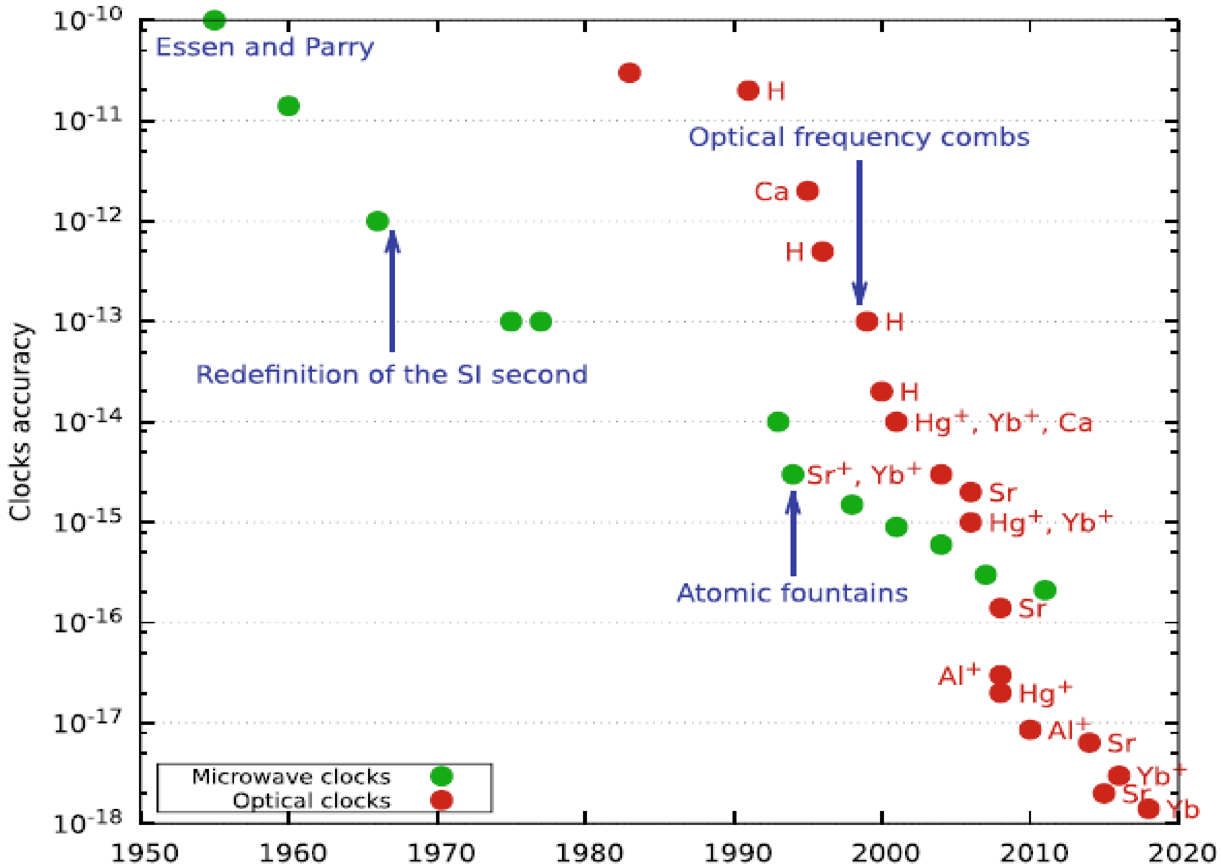
- Clock with stability 10^{-18} corresponds to $\Delta W \approx 0.1\ m^2/s^2$, and about ≈ 1 cm in physical heights

$$H_B = \frac{H_A \cdot \bar{g}_A - \Delta W_{AB}}{\bar{g}_B} \quad (11)$$



2. Realization of Arab Height Reference System

Accuracy Records for atomic Clocks

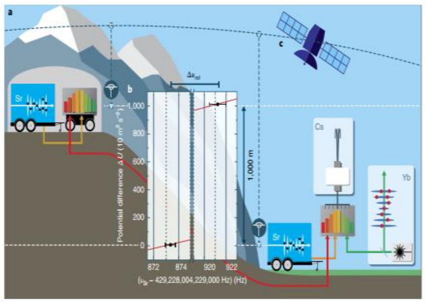


Stability of Microwave and Optical Clocks (Delva et al. 2019)

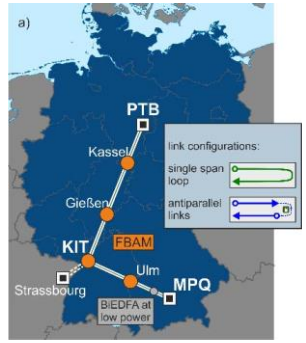
Challenge: Not only atomic clocks with **high stability** but also reliable time/frequency **transfer links**.

2. Realization of Arab Height Reference System

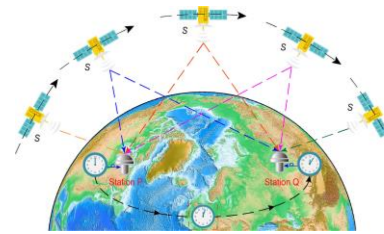
- ## Time – Frequency Methods for Clock Comparison



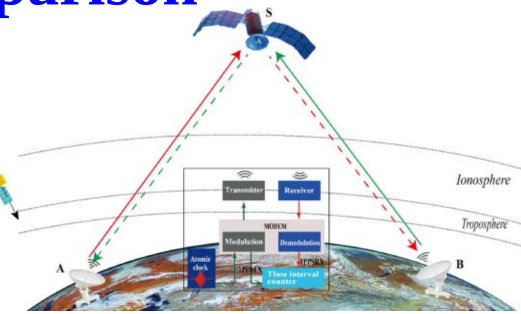
Clock Transportation
(Grotti et al 2018)



Optical Fiber Link
(A.. Kuhl et al. 2019)

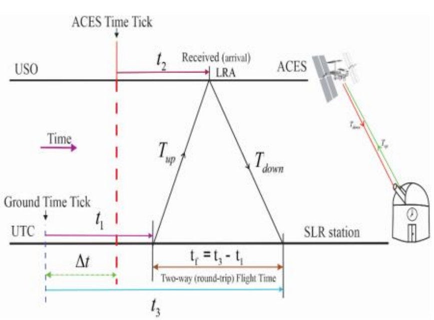


GNSS PPP Time Transfer Technique (Cai et al. 2020)

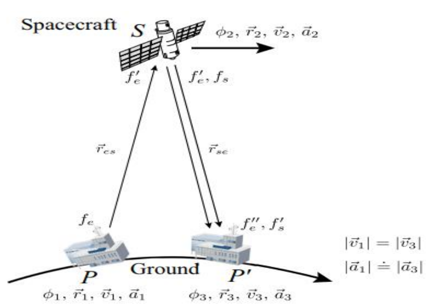


TWSTFT Time Transfer Technique
(Wu et al. 2020)

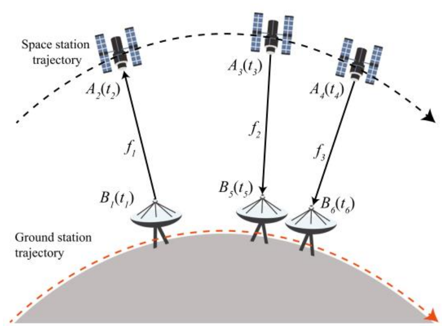
Most Potentially Perspectives



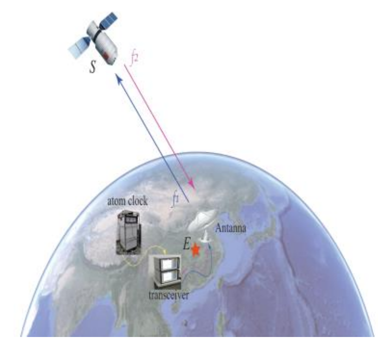
Two-Way Laser Time Transfer Link
(Ruby et al. 2021)



Satellite Frequency Signal Transmission
(SFST) (Shen et al. 2019)



Tri-frequency combination Link
(Sun et al. 2021)

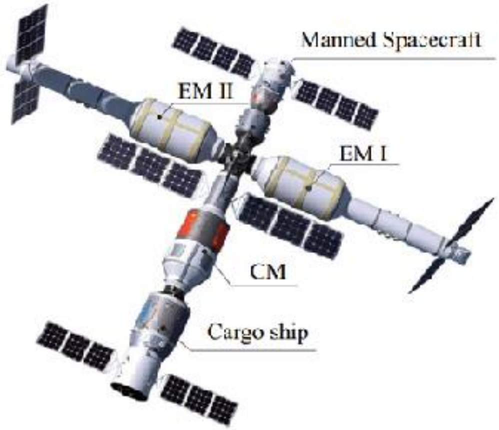


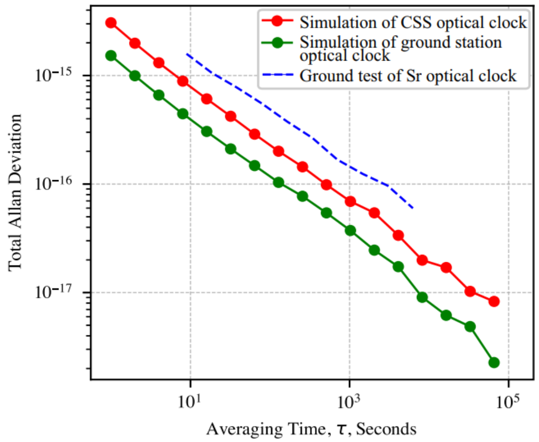
Two-frequency combination Link (Shen et al. 2023, PRD)

2. Realization of Arab Height Reference System

▪ Future Mission for Clock Comparison

- China Space Station (CSS) was launched in October 2022

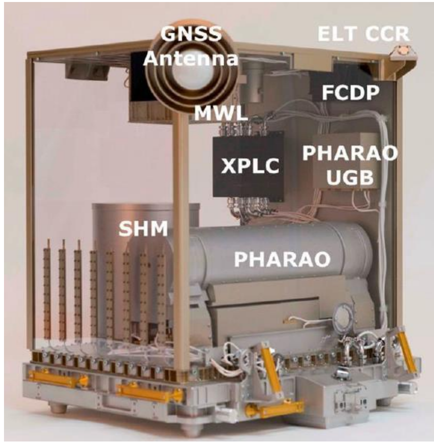


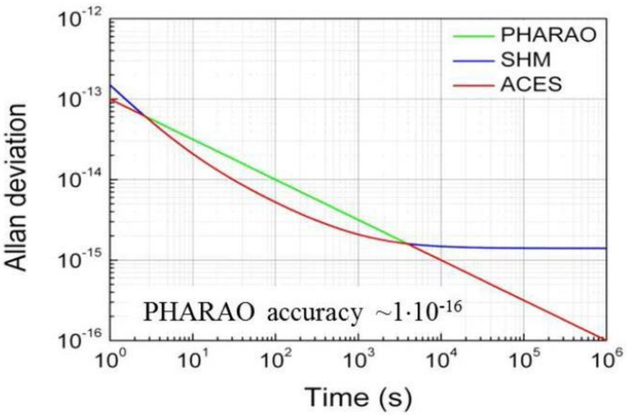


Accuracy of Clocks
$\sim 8 \times 10^{-18}$

(shen et al., 2023, PRD)

- Atomic Clock Ensemble in Space (ACES) is planned for 2025





Accuracy of Clocks
$\sim 1 \times 10^{-16}$

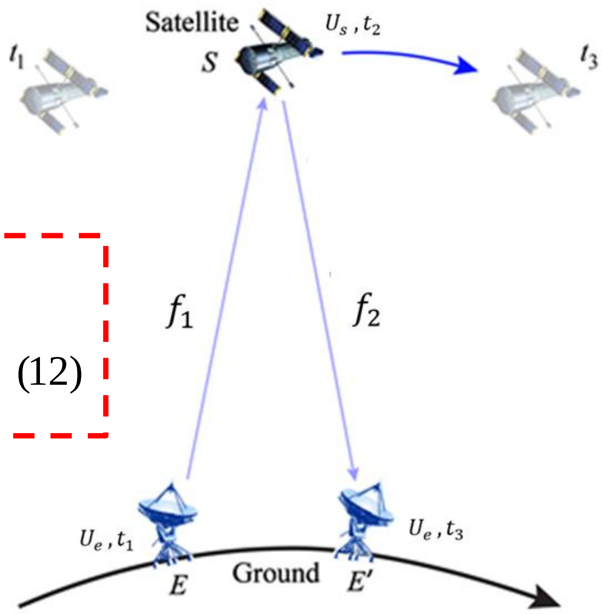
(Cacciapuoti et al., 2017)

2. Realization of Arab Height Reference System

▪ Arab Height System and Two-Way Frequencies Link

- Up-link and down-link signals between satellite (denoted as S) and ground station (denoted as E) satisfy the following expression:

$$f_{ES}^{r} = f_{ES}^{e} - \Delta U_{ES} + \left(\Delta f_{Doppler1} + \Delta f_{Doppler2}\right)_{ES} + \Delta f_{Troposphere-ES}$$
$$+ \Delta f_{ionosphere-ES} - \left(\Delta f_{Tides} + \Delta f_{External\ Mass}\right)_{ES} + \Delta f_{Relativistic-ES} + \varepsilon_{ES} \quad (12)$$



- A combination of two frequencies link, the gravitational potential (GP) difference is given by:

$$\Delta U_{SE} = \frac{f_{SE}^{r} - f_{ES}^{r}}{2} + \Lambda f_{ES} + \varepsilon \quad (13)$$

Details in Shen et al. (2023, PRD)

$\Delta U_{ES} = U_S - U_E$, GP difference

Λf: Sum of all correction terms (such as ionosphere and tropospheric effects…..etc.).

ε : Un-modeled errors of signals (clock errors, random errors and other noises).

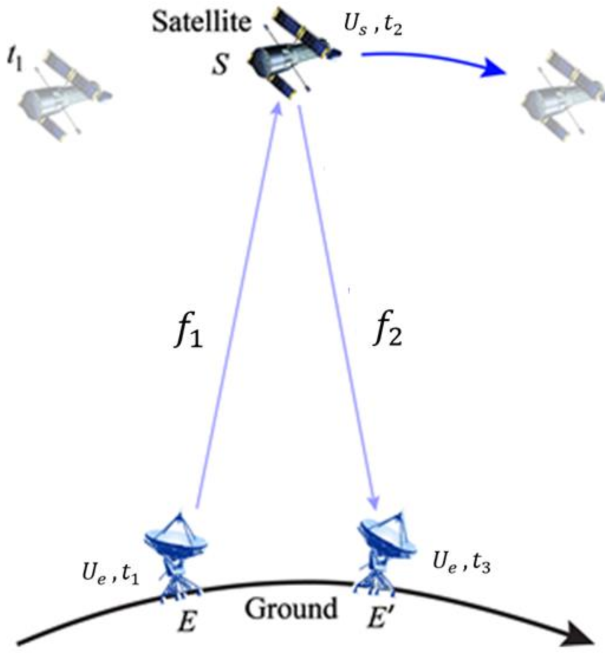
2. Realization of Arab Height Reference System

▪ Arab Height System and Two-Way Frequencies Link

- We focus on geopotential difference between satellite and ground as a time series, which is the sum of gravitational potential (*U*) and centrifugal potential (Z), expressed as:

$$\Delta W_{ES}(t) = \Delta U_{\text{ES}}(t) + \Delta Z_{\text{ES}}(t) \qquad (14)$$

Where : $$Z = \frac{1}{2}\omega^2(X^2 + Y^2)$$



- For any two arbitrary points on geoid, the geopotential difference can be measured as a time series:

$$\Delta W_{AB}(t) = W_A - W_B = \Delta U_{\text{A-Satellite}}(t) - \Delta U_{\text{B-Satellite}}(t) + (Z_A - Z_B) \qquad (15)$$

- Suppose height of point ***A*** is given, then height of point ***B*** can be obtained as:

$$H_B = \frac{H_A \cdot \bar{g}_A - \Delta W_{AB}}{\bar{g}_B} \qquad (16)$$

(Shen et al. 2023)

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3. Simulation Experiments

Unify Height System between Egypt and Kingdom of Saudi Arabia

Experiment setup

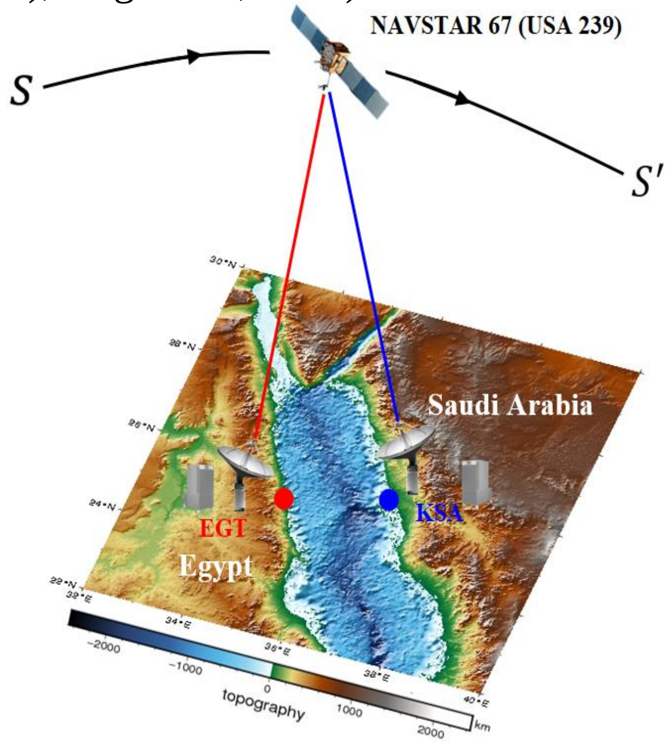
- Experiments are conducted at the time duration when the satellite USA 239 moves from position S to position S' (from UTC 09:00 PM to 12:00 PM (3 hours), August 30, 2023).

Table (1): Details of selected stations.

Country	Station Name	World Geodetic System 1984		Vertical Reference Frame	
		Latitude [DD.ddd]	Longitude [DD.ddd]	Orthometric Height [m]	Datum
Kingdom of Saudi Arabia (KSA)	TB05	26.366944	36.368639	17.533	KSA-VRF14
Egypt (EGT)	OZ09	26.017434	34.321205	11.2636	EGT-VD06

Table (2): Parameters of Simulation and frequency links.

Items	Values of Parameter
Satellite Name	NAVSTAR 67 (USA 239: GPS Navigation Sat.)
Altitude	~ 20228 Km
Orbit inclination	53.5 °
Position/ Velocity accuracy of satellite	0.1 m and $1 \times 10^{-5}\ m/s$
On-board clock stability	$5 \times 10^{-16}/\sqrt{\tau}$
Ground clock stability	$5 \times 10^{-17}/\sqrt{\tau}$
Uplink frequency	1.575 GHZ
Downlink frequency	1.575 GHZ
Observation duration	from 9:00 PM to 12:00 PM (3 h), August 30, 2023
Observation cutoff elevation angle	15°
Measurement interval	1 s (No. of observation = 10800)
Gravity field model	EGM2008
Ionospheric model	International Reference Ionosphere
Tropospheric model	Earth Global Reference Atmospheric Model
Tide correction	ETERNA
Height system difference	6.490 m

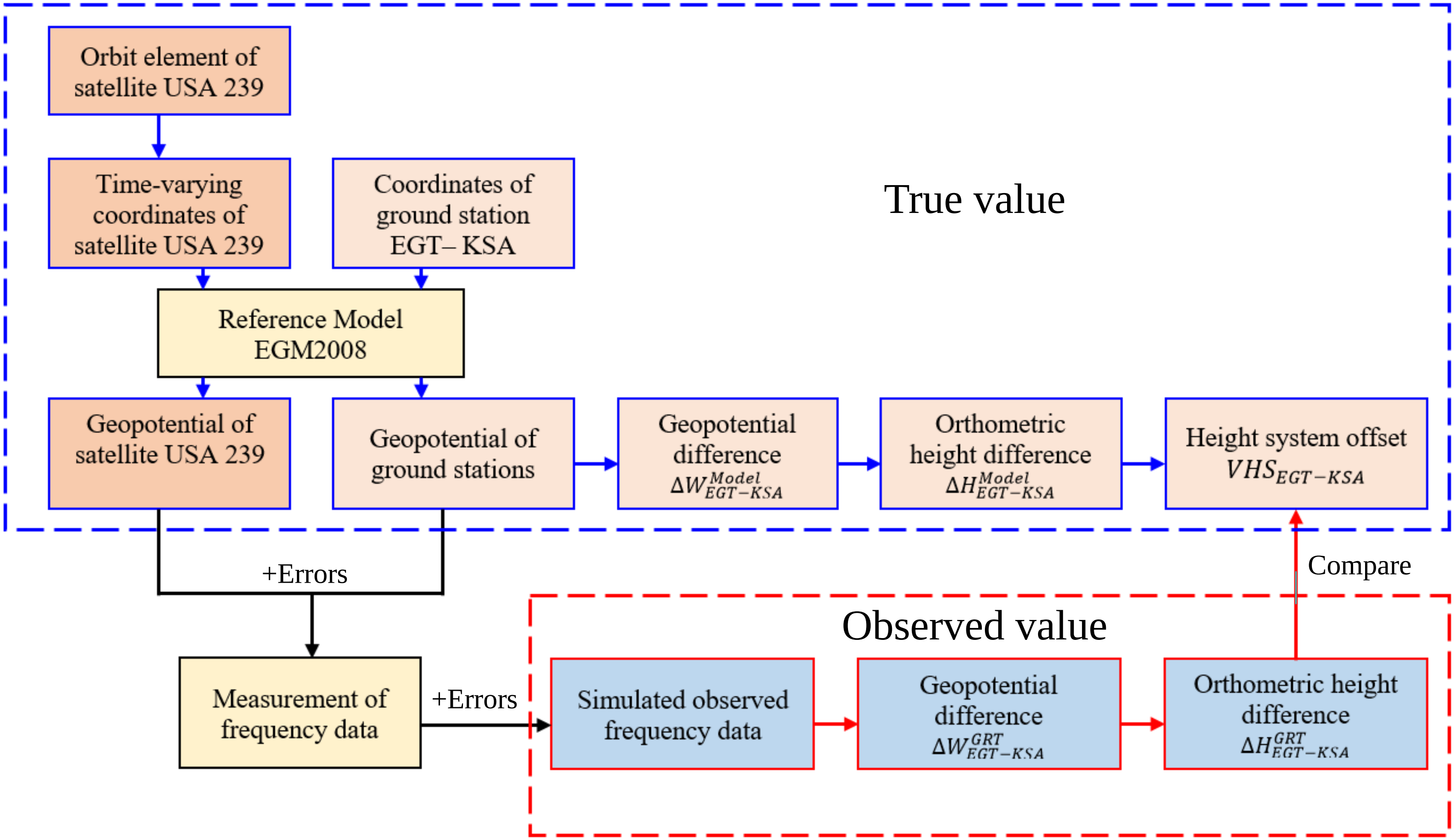


Connection of Egypt (EGT) datum and Kingdom of Saudi Arabia (KSA) datum via two-way frequency link

3. Simulation Experiments

- **Unify Height System between Egypt and Kingdom of Saudi Arabia**

Scheme of simulation experiment

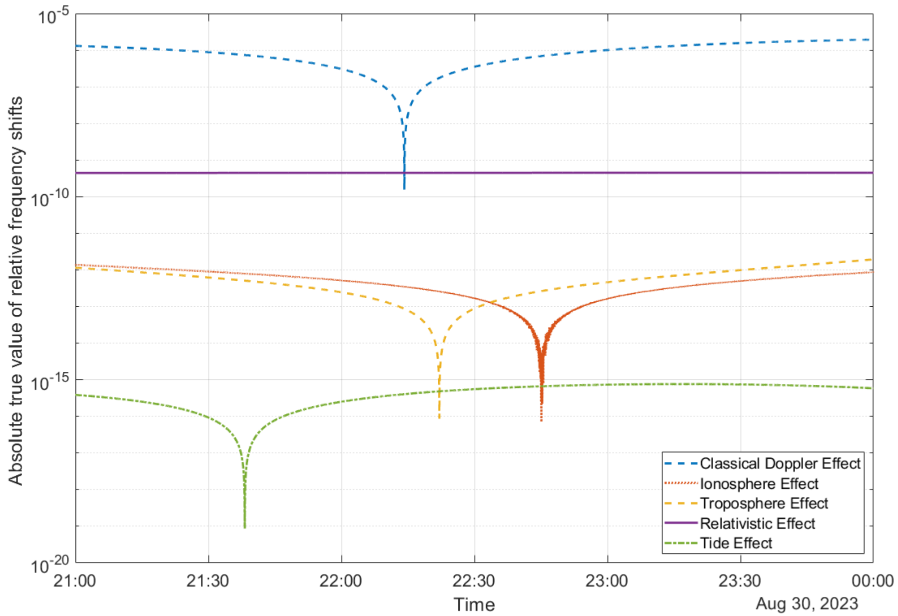


3. Simulation Experiments

- **Unify Height System between Egypt and Kingdom of Saudi Arabia**

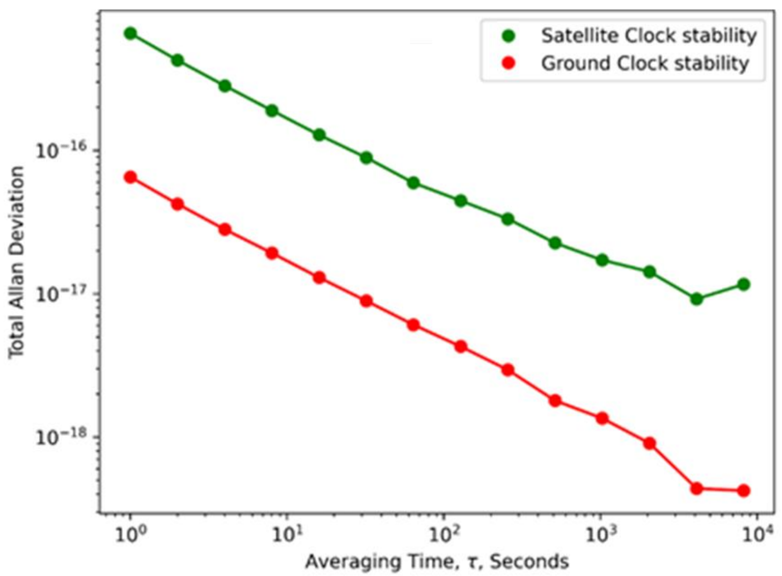
Results and Analysis

Relative Frequency Shifts in One-way Frequency Link.



- Different colored curves denote different relative frequency shifts in a one-way transfer.

Stability of Clocks' Frequency Signals.



- The red and green lines are simulation clock error data for satellite and ground clock, and the stabilities reach $5 \times 10^{-16}/\sqrt{\tau}$ and $5 \times 10^{-17}/\sqrt{\tau}$ (τ in second), respectively.

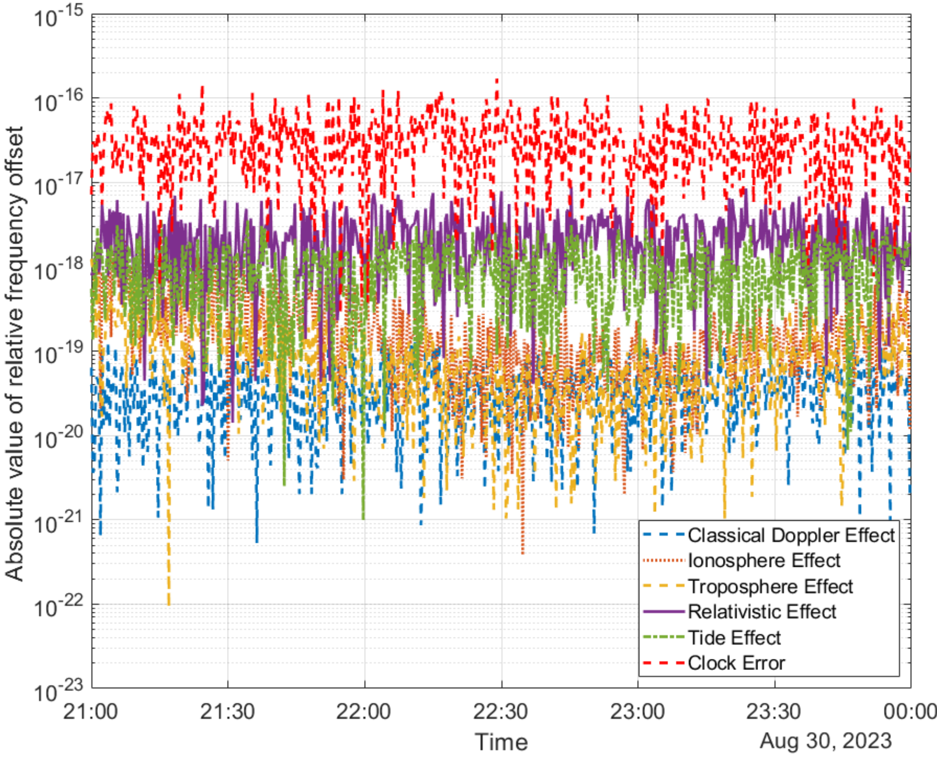
3. Simulation Experiments

- **Unify Height System between Egypt and Kingdom of Saudi Arabia**

Results and Analysis

- Error sources and their magnitudes in determining the gravitational potential difference between a satellite and a ground station.

Error Sources of Various Frequency Shifts	Residual errors
Classical Doppler Effect	5.26×10^{-20}
Ionosphere shift Error	3.74×10^{-19}
Troposphere shift Error	1.91×10^{-19}
Relativistic Effect	2.92×10^{-18}
Tide shift Error	1.27×10^{-18}
Clock Error	4.31×10^{-17}



- The two-way frequency method can mostly eliminate various errors. Only, the stability of the clocks will affect the results.

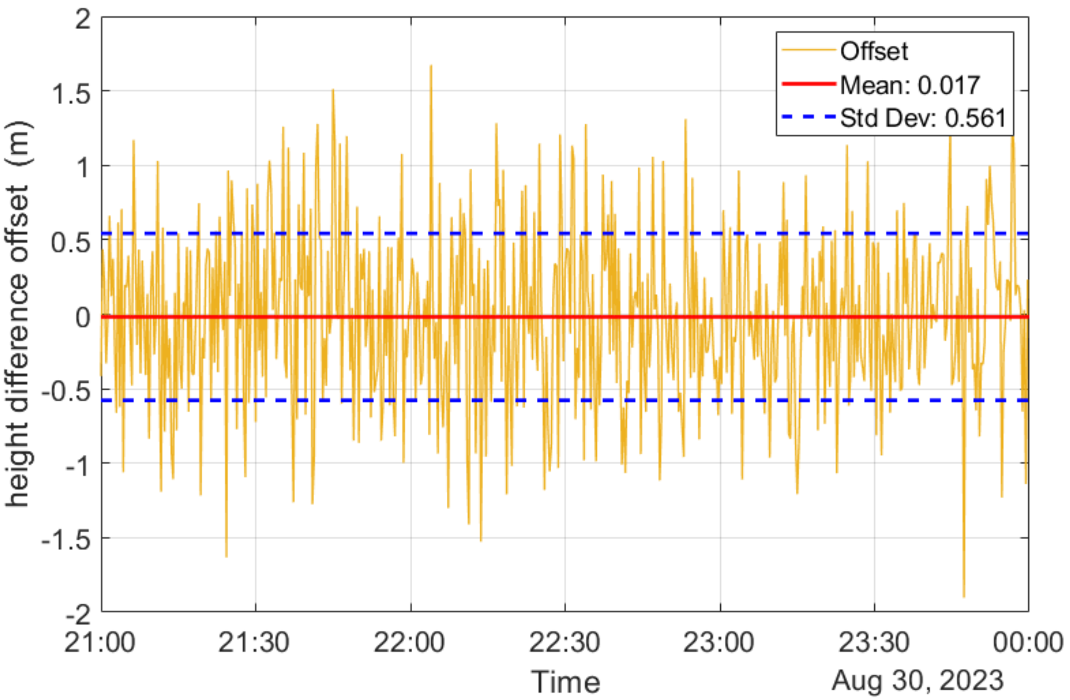
3. Simulation Experiments

- **Unify Height System between Egypt and Kingdom of Saudi Arabia**

Results and Analysis

- We can see that the mean offset was at the centimeter level, while STD was at the decimeter level, because the clock stability is $\mathbf{5 \times 10^{-17}/\sqrt{\tau}}$ corresponding to **0.5 m in height**.

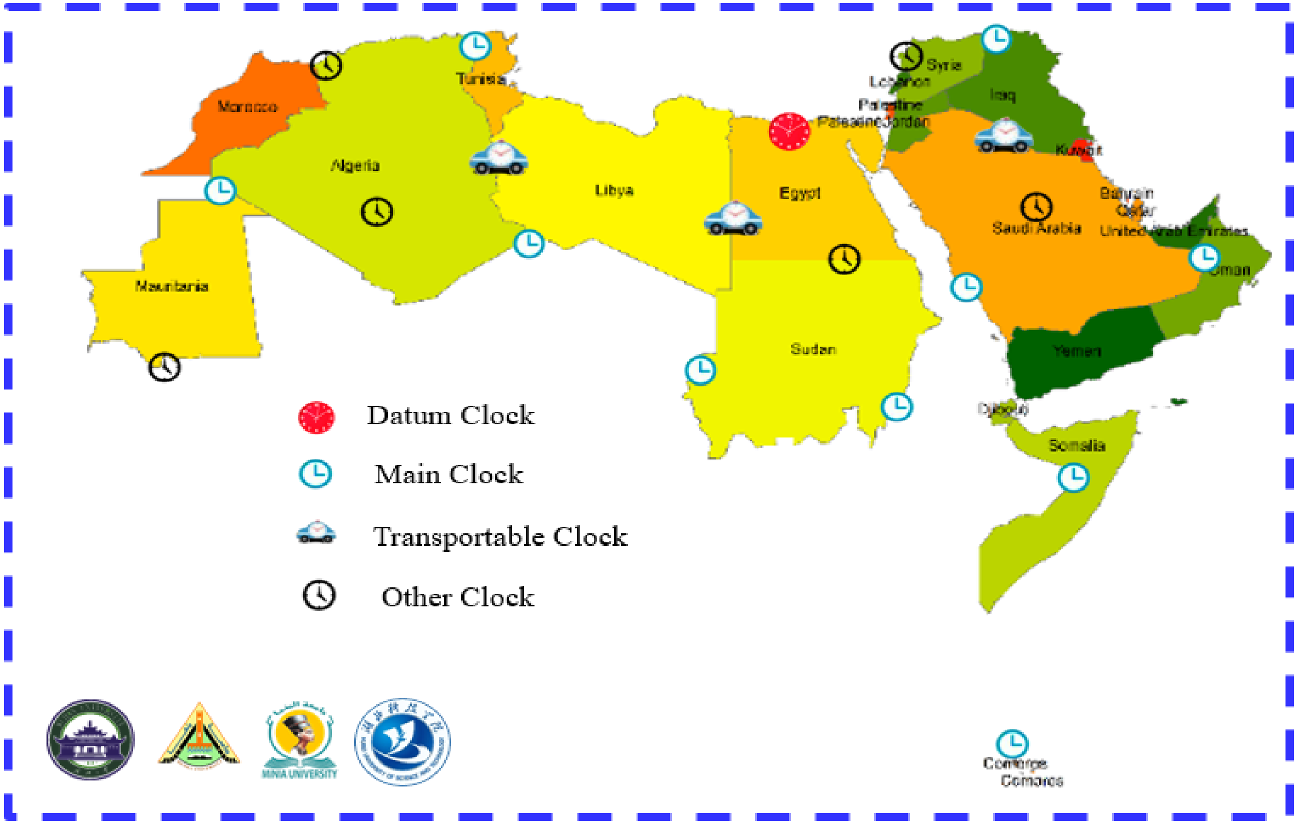
- To estimate the reliability of the two-way frequency method, we will conduct different simulation experiments with **different chosen parameters**.



The offset between true values and estimated values of height datum difference

3. Simulation Experiments

- **Next step: We will start a project “ Clocks Network for Arab Height Reference System (AHRS) Unification”**



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4. Conclusions

- A unified height system can **contribute to economic** growth by reducing project costs, minimizing errors, and improving the accuracy of geographic information.
- Arab Height Reference System (AHRS) **should be consistent** with the International Height Reference System (IHRS) or $\delta W = W_0^{IHRS} - W_0^{Local}$
- The two-way frequency combination method is very promising for unifying the Arab World's vertical datum **at the cm level**.
- **UN-GGIM:** Encourage Arab countries to **share** existing height data and **work together** to standardize measurement techniques, reference frames, and data formats. This will be a **fundamental step** in achieving unification.

Thanks for your Attention

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